Brane World Linearized Cosmic String Gravity

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Abstract

The gravitational properties of cosmic strings in the Randall-Sundrum brane world scenario are investigated. Using a gauge in which the brane remains straight, the leading order corrections to the metric on the brane are determined. In contrast to their non-brane equivalents, these strings have an attractive $1/r^2$ potential and a radially dependent deficit angle. These two effects alter many cosmological properties of the string, such as the formation of double images. As a result of the attractive force the string will collect matter as it moves through space.

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1 Introduction

Cosmic strings are a type of topological defect formed at phase transitions in some grand unified theories (GUTs) [1]. In the past their gravitational effects have been considered as a possible explanation for the anisotropies in the cosmic microwave background (CMB). Recent observations [2] and simulations [3] have raised doubts about this.

It has been suggested that the physical universe may be a 3-brane embedded in a higher dimensional spacetime [4]. Ordinary matter is confined to this brane, while gravity is free to propagate in the higher dimensional bulk. To be phenomenologically credible, gravity on the brane must, at least at large distances, resemble 4-dimensional Einstein gravity. In a scenario proposed by Randall and Sundrum the bulk space has a non-zero cosmological constant. This gives a mass to the bulk gravitons, suppressing their effects. The standard 3+1-dimensional gravity is produced by massless gravitons trapped on the brane, with small corrections from the bulk. By tuning the brane tension the effects of the bulk cosmological constant can be cancelled on the brane.

In contrast to a point particle, a cosmic string (in the standard 4-dimensional universe) does not exert a force on nearby matter or radiation. Its gravitational effects result from the the fact that the space around it is approximately conical. If two initially parallel particle paths pass either side of the string, the conical geometry will cause their paths to converge. A related effect is the formation of double images. The string's gravitational lensing means that if an observer is on the opposite side of a cosmic string to a quasar, he will see two images of it [6, 7].

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The conical geometry also produces a Doppler shift in light emitted from objects moving relative to the string (assuming the string is between the object and the observer). When a string passes between an observer and the cosmic microwave background this Doppler effect will produce anisotropies [8, 7]. It is the CMB anisotropies predicted by this effect which have conflicted with observational data, and so damaged the credibility of cosmic string models.

In this paper the gravitational field around a brane world cosmic string is found, using a weak field approximation. The resulting field is significantly different to that of a string in the standard four dimensional universe. This means that any cosmological properties of the string which arise from its gravity need to be revised in the brane world scenario. This is in contrast to point-like particles, where the brane world gravity is qualitatively the same as in the standard case. The geodesics away from the string are also determined. These are used to give a modified expression for the separation of double images created by the string.

2 Linearized Brane Gravity

We will start by reviewing a formulation of linearized gravity in which the brane is kept straight, even when matter is present [9]. The metric can be written as

$$ds^2 = \hat{g}_{\mu\nu} dx^\mu dx^\nu + 2n_\mu dx^\mu dy + (1+\phi)dy^2 \tag{1}$$

with

$$\hat{g}_{\mu\nu} = e^{-2|y|/\ell}(\eta_{\mu\nu} + h_{\mu\nu}) \tag{2}$$

x^μ, with $\mu = 0, 1, 2, 3$, are the coordinates on the brane and the fifth coordinate y is chosen so that the brane is at $y = 0$. $\hat{g}_{\mu\nu}$ gives the induced metric on the hypersurfaces $y =$ constant. The length ℓ is related to the bulk cosmological constant by $\Lambda = 6/\ell^2$. ℓ gives the characteristic length scale of the brane corrections to the usual 3+1-dimensional Einstein gravity. I will use the four dimensional Minkowski metric $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$.

In the linearized approximation $h_{\mu\nu}$, n_μ and ϕ are taken to be small. In this approximation the gauge conditions for the brane to remain at $y = 0$ are

$$\phi = -\frac{\ell}{4}\,\mathrm{sgn}\, y\, h_{,y} \tag{3}$$

$$n_\mu = -\frac{\ell}{8}\,\mathrm{sgn}\, y\, h_{,\mu} \tag{4}$$

$$\tilde{h}_{\mu\nu}{}^{,\mu} = 0 \tag{5}$$

where $h = \eta^{\mu\nu} h_{\mu\nu}$ and $\tilde{h}_{\mu\nu} = h_{\mu\nu} - (1/4)\eta_{\mu\nu} h$ is the traceless part of $h_{\mu\nu}$.

The energy momentum tensor is

$$T^{\mu\nu} = -\frac{3}{4\pi\ell}\sqrt{1-\phi}\,\hat{g}^{\mu\nu}\delta(y) + t^{\mu\nu}\delta(y)\ , \quad T^{5\mu} = T^{55} = 0 \tag{6}$$

The first term is the background from the brane, and $t^{\mu\nu}$ is the matter perturbation on the brane. The resulting Einstein equations (to leading order) are

$$\partial_y\left(e^{-2|y|/\ell}\partial_y \tilde{h}_{\mu\nu}\right) - \frac{2}{\ell}\,\mathrm{sgn}\, y\, e^{-2|y|/\ell}\partial_y \tilde{h}_{\mu\nu} + \partial^\sigma\partial_\sigma \tilde{h}_{\mu\nu} = G_5\delta(y)\left[-16\pi\left(t_{\mu\nu} - \frac{1}{3}\eta_{\mu\nu} t\right) - \frac{\ell}{2} h_{,\mu\nu}\right] \tag{7}$$

where $t = \eta^{\mu\nu} t_{\mu\nu}$. G_5 is the 5-dimensional Newton's constant. It is related to the fundamental 5-dimensional Planck mass M_5 by $G_5 = M_5^{-3}$. The effective 4-dimensional Newton's constant is $G_4 = M_4^{-2} = \ell^{-1} G_5$. Gravity experiments constrain ℓ to be less than 1 mm, and so $M_5 \gtrsim 10^5$ TeV.

Taking the divergence of (7) and using (5) gives the constraint

$$\partial^\sigma \partial_\sigma h|_{y=0} = \frac{32\pi}{3\ell} t \, . \tag{8}$$

In momentum space (7) has the solution

$$\tilde{h}_{\mu\nu}(p, y) = 8\pi G_5 \left[t_{\mu\nu} - \frac{1}{3} \left(\eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) t \right] e^{2|y|/\ell} \frac{K_2(e^{|y|/\ell} p\ell)}{pK_1(p\ell)} \tag{9}$$

for $p^2 > 0$.

3 Cosmic String Gravity

The cosmic string metric on the brane can now be found by substituting the string energy-momentum tensor into (9) and setting $y = 0$. For simplicity I will take the limit of zero string width. In this case its energy momentum is [6]

$$t_{\mu\nu} = \mu \delta^2(x) \mathrm{diag}(1, 0, 0, -1) \tag{10}$$

thus $t = -2\mu\delta^2(x)$. μ is the energy per unit length of the string. It is proportional to the square of the symmetry breaking scale at which the string formed. Note that since the string is 'made' of Higgs and gauge fields, it is confined to the brane and does not extend into the fifth, bulk dimension. Substituting (10) into (9) and inverting the Fourier transforms gives

$$\tilde{h}_{33}(x, 0) = -\tilde{h}_{00}(x, 0) = -\frac{8\pi}{3} G_5 \mu \int \frac{K_2(p\ell)}{pK_1(p\ell)} e^{i\mathbf{p}\cdot\mathbf{x}} \frac{d^2p}{(2\pi)^2} \tag{11}$$

$$\tilde{h}_{ij}(x, 0) = \frac{16\pi}{3} G_5 \mu \int \left(\delta_{ij} - \frac{p_i p_j}{p^2} \right) \frac{K_2(p\ell)}{pK_1(p\ell)} e^{i\mathbf{p}\cdot\mathbf{x}} \frac{d^2p}{(2\pi)^2} \tag{12}$$

with $i, j = 1, 2$ and $\mathbf{p} = (p_1, p_2)$, $\mathbf{x} = (x_1, x_2)$. Changing to cylindrical polar coordinates, (12) implies

$$\tilde{h}_{rr}(x, 0) = \frac{16\pi}{3} G_5 \mu \int \sin^2(\theta - \beta) \frac{K_2(p\ell)}{pK_1(p\ell)} \left(\sum_n i^n J_n(pr) e^{in(\theta-\beta)} \right) \frac{pdpd\beta}{(2\pi)^2} \tag{13}$$

where $\mathbf{p} = p(\cos\beta, \sin\beta)$ and $\mathbf{x} = r(\cos\theta, \sin\theta)$. Evaluating the angular integral simplifies (13) to

$$\tilde{h}_{rr}(r, 0) = \frac{4}{3} G_5 \mu \int \frac{K_2(p\ell)}{pK_1(p\ell)} [J_0(pr) + J_2(pr)] \, pdp \tag{14}$$

Similarly

$$\tilde{h}_{\theta\theta}(r, 0) = r^2 \frac{4}{3} G_5 \mu \int \frac{K_2(p\ell)}{pK_1(p\ell)} [J_0(pr) - J_2(pr)] \, pdp \tag{15}$$

$$\tilde{h}_{33}(r, 0) = -\tilde{h}_{00}(r, 0) = -\frac{4}{3} G_5 \mu \int \frac{K_2(p\ell)}{pK_1(p\ell)} J_0(pr) \, pdp \tag{16}$$

Using (8) gives

$$h(r,0) = \frac{16}{3} G_5 \mu \int \frac{2}{p^2 \ell} J_0(pr)\, pdp \tag{17}$$

Combining this with (16) and simplifying gives

$$h_{33}(r,0) = -h_{00}(r,0) = -\frac{4}{3} G_5 \mu \int \frac{K_0(p\ell)}{pK_1(p\ell)} J_0(pr)\, pdp \tag{18}$$

The large r behaviour of $h_{\mu\nu}$ could now be found by expanding the above integrands around $p = 0$ and evaluating the integrals. Doing this gives $h_{\mu\nu} \sim \ln r$ for large r, suggesting that the weak field approximation breaks down. A similar situation arises around the standard non-brane cosmic strings [6]. In fact this problem is just the result of a poor choice of coordinates.

A better behaved solution is obtained by introducing a new radial coordinate r' satisfying

$$\left[1 + \frac{4}{3} G_4 \mu \int \frac{\ell K_2(p\ell)}{pK_1(p\ell)} [J_0(pr) - J_2(pr)] + \frac{2}{p^2} J_0(pr)\, pdp\right] r^2 = \left[1 - 8G_4\mu\{1 + f_1(r'/\ell)\}\right] r'^2 \tag{19}$$

where f_1 is the solution of the inhomogeneous differential equation

$$\partial_x [x f_1(x)] = \frac{1}{3} \int \frac{K_0(q)}{K_1(q)} x J_1(qx)\, qdq \tag{20}$$

with boundary condition $x f_1(x)|_{x\to\infty} = 0$.

Combining (14,15,17) and (18) with (19), we find to leading order in $G_4\mu$ that the metric is

$$ds^2 = (-dt^2 + dz^2)[1 - G_4\mu f_2(r'/\ell)] + dr'^2 + [1 - 8G_4\mu\{1 + f_1(r'/\ell)\}] r'^2 d\theta^2 \tag{21}$$

with

$$f_2(x) = \frac{4}{3} \int \frac{K_0(q)}{qK_1(q)} J_0(qx)\, qdq \tag{22}$$

The metric for a cosmic string in the standard four dimensional universe is obtained from (21) by replacing f_1 and f_2 with zero. The resulting geometry is conical. Thus, outside the string core, spacetime is locally flat.

For small q

$$\frac{K_0(q)}{qK_1(q)} \approx -\ln(q/q_*) + \frac{1}{2}\left(\ln^2(q/q_*) - \ln(q/q_*) + \frac{1}{2}\right) q^2 + O(q^4 \ln^2 q) \tag{23}$$

with $q_* = 2e^{-\gamma}$, where γ is Euler's constant. Using this approximation (20) and (22) are solved by

$$f_1(r/\ell) \sim -\frac{2\ell^2}{3r^2}\left[1 - \frac{2\ell^2}{9r^2}\{12\ln(r/\ell) - 5\} + \cdots\right] \tag{24}$$

$$f_2(r/\ell) \sim \frac{4\ell^2}{3r^2}\left[1 - \frac{2\ell^2}{r^2}\{2\ln(r/\ell) - 1\} + \cdots\right] \tag{25}$$

For brevity I have dropped the prime on r'. The above asymptotic expansions are valid for $r \gg \ell$.

To leading order in $G_4\mu$ and ℓ/r the metric (21) is thus

$$ds^2 = (-dt^2 + dz^2)\left[1 - \frac{4G_4\mu\ell^2}{3r^2}\right] + dr^2 + \left[1 - 8G_4\mu + \frac{16G_4\mu\ell^2}{3r^2}\right] r^2 d\theta^2 \tag{26}$$

Thus space is approximately conical for $r \to \infty$, with the same deficit angle as the standard 4-dimensional cosmic string spacetime [6]. The brane corrections give the string a small $1/r^3$ attractive force, and a deficit angle which increases with r.

The approximation (10) assumes that all length scales of interest are far bigger than the string width, $r_{\rm s} \sim \mu^{-1/2}$. The GUT energy scale must be less than the five dimensional Planck scale, so it may be lower than in the standard four dimensional universe. Unless μ is far lower than M_5^2 (which is not to be expected for a GUT string), $r_{\rm s} \ll \ell$. In this case the solution (21) will still be valid for small r. For $r_{\rm s} \ll r \ll \ell$ series expansions of (20) and (22) can be found using the asymptotic expansion

$$\frac{K_0(q)}{qK_1(q)} \sim \frac{1}{q} - \frac{1}{2q^2} + O(q^{-3}) \tag{27}$$

This implies

$$f_1(r/\ell) \approx \frac{\ell}{3r}\ln(r/\ell) - \frac{1}{6} + \cdots \tag{28}$$

$$f_2(r/\ell) \approx \frac{4\ell}{3r}\left[1 + \frac{r}{2\ell}\ln(r/\ell)\right] + \cdots \tag{29}$$

For $r \sim G_4\mu\ell = G_5\mu$ the linearized gravity approximation will break down. Unless $\mu \sim M_5^2$ this will occur inside the string core. Thus the above linearized string gravity will usually be valid everywhere outside the string core. Note that as the string is approached, the gravitational field does genuinely become stronger, in contrast to the previous $\ln r$ behaviour at large r (see comments before eq. 19), which was purely due to a poor choice of coordinates. This can be seen from the behaviour of the Ricci tensor as $r \to 0$ (see appendix).

Far away from the string the gravitational field is very weak. If $\ell \sim 1\,\mathrm{mm}$, then for a typical GUT-scale string with $G_4\mu \sim 10^{-6}$, the gravitational acceleration at $r \sim 1\,\mathrm{pc}$ will be of order $10^{-42}\,\mathrm{cm\,s^{-2}}$. Thus the string's gravitational effects will not be significant at astronomical distances.

4 Geodesics in a Cosmic String Background

Using (26) particle paths at large r can be found. In calculating the geodesic equations we must use the intrinsic connection coefficients $\hat{\Gamma}^{\mu}{}_{\nu\lambda}$ (see appendix), instead of the full 5-dimensional equivalents $\Gamma^{\mu}{}_{\nu\lambda}$. This is because ordinary matter is confined to the brane. If a particle were free to move in five dimensions the usual geodesic equations (with $\Gamma^{\mu}{}_{\nu\lambda}$) would apply.

To leading order in $G_4\mu$, particle paths are given by

$$\dot{t} = E[1 + G_4\mu f_2(r/\ell)]\ , \qquad \dot{z} = P_z[1 + G_4\mu f_2(r/\ell)] \tag{30}$$

$$\dot{\theta} = \frac{J}{r^2}[1 + 8G_4\mu\{1 + f_1(r/\ell)\}] \tag{31}$$

$$\dot{r}^2 - G_4\mu f_2(r/\ell)(P_T^2 + m^2) + \frac{J^2}{r^2}[1 + 8G_4\mu\{1 + f_1(r/\ell)\}] = P_T^2 \tag{32}$$

The dot denotes differentiation with respect to the geodesic's parameter. If this parameter is taken to be τ/m, where τ is the proper time, then E, J, P_z and P_T are respectively the

energy and angular, z, and transverse momentum at $r = \infty$. m is the particle mass, so $P_T^2 = E^2 - P_z^2 - m^2$.

Combining (31) and (32), and keeping only the leading order (in ℓ/r) terms of (24) and (25) produces

$$\left(\frac{\partial r}{\partial \theta}\right)^2 = \frac{P_T^2}{J^2}(1 - 16G_4\mu)r^4 - \left(1 - 8G_4\mu - \frac{4(9P_T^2 + m^2)G_4\mu\ell^2}{3J^2}\right) r^2 - \frac{16}{3}G_4\mu\ell^2 \tag{33}$$

The $O(G_4\mu)^2$ terms in the above expression have been dropped.

With the change of variables $w = \alpha/r$ and $\theta = \pm\beta u$, (33) can be rewritten as

$$\frac{dw}{du} = -\sqrt{(1 - w^2)(1 + k^2[w^2 - 1])} \tag{34}$$

where k is a suitably chosen constant. This equation is solved by the Jacobian elliptic function $w = \operatorname{cn}(u, k)$ [10]. Thus

$$r = \frac{\alpha}{\operatorname{cn}(\theta/\beta, k)} \tag{35}$$

The function $\operatorname{cn}(u, k)$ is periodic and can be thought of as a generalisation of $\cos u$. It has period $4K$, where K is the complete elliptic integral $\int_0^{\pi/2} da(1 - k^2 \sin^2 a)^{-1/2}$. The function $1/\operatorname{cn}$ can be represented as a trigonometric series involving terms proportional to $\sec \pi u/(2K)$ and $\cos(2n - 1)\pi u/(2K)$ (for integer n). If k is small $K = (\pi/2)[1 + k^2/4 + O(k^4)]$ and

$$\frac{1}{\operatorname{cn}(u, k)} = \frac{\pi}{2K} \sec \frac{\pi}{2K}u + O(k^2) \tag{36}$$

To leading order in $G_4\mu$

$$\beta = 1 + 4G_4\mu + \frac{2(P_T^2 + m^2)}{3J^2}G_4\mu\ell^2 + O(G_4\mu)^2 \tag{37}$$

$$k^2 = \frac{16P_T^2}{J^2}G_4\mu\ell^2 + O(G_4\mu)^2 \tag{38}$$

$$\alpha = \frac{J}{P_T}\left(1 + 4G_4\mu - \frac{2(5P_T^2 + m^2)}{3J^2}G_4\mu\ell^2\right) \tag{39}$$

Thus unless $P_T\ell \gg J$, the parameter k is small. Substituting (36–38) into (35) gives the solution for r (to leading order in $G_4\mu$)

$$r = \frac{J}{P_T} \sec \frac{\theta}{1 + \epsilon} \tag{40}$$

where

$$\epsilon = 4G_4\mu + \frac{2(3P_T^2 + m^2)}{3J^2}G_4\mu\ell^2 + O(G_4\mu)^2 \tag{41}$$

This solution represents a particle approaching the string from $\theta = \frac{\pi}{2}(1 + \epsilon)$ and leaving at $\theta = -\frac{\pi}{2}(1 + \epsilon)$. Thus the particle path is deflected by an angle of $\pi\epsilon$. The corresponding result for a non-brane string is obtained by setting $\ell = 0$.

If $k > 1$ (34) will instead be solved by $w = \operatorname{dn}(ku, k^{-1})$ (another Jacobian elliptic function). For small k, $\operatorname{dn}(u, k)$ is approximately a constant plus small oscillatory terms proportional to $\cos n\pi u/K$ (integer n). This solution thus represents a stable orbit around the

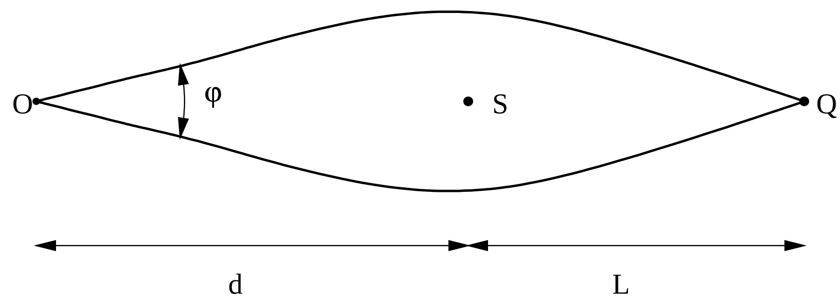

Figure 1: Double image of an object behind a string

string. Normally a system with a $-1/r^2$ potential would have orbiting solutions, but they would not be stable. However, the spatially varying deficit angle of the string produces an additional $1/r^4$ effective potential term in (32), which stabilises them. Note that these forces do not affect motion of particles in the z-direction, thus this type of solution includes helical as well as circular paths.

If k is to be large, J must be smaller than $P_T\ell\sqrt{G_4\mu}$ [see (38)]. In this case the f_1 term in (32) required to stabilise the orbit is of similar size to terms that have been dropped in the linearized gravity approximation. The stability of these orbits may therefore disappear when higher order terms are considered. The orbiting particles will then either escape to $r = \infty$ or fall into the string core, where (since GUT processes are enhanced inside the string) they are expected to decay into lighter fundamental particles or string bound states. Whether the orbits are stable or not, the string will collect matter as it passes through plasma, with the trapped matter ending up in the string core or orbiting it.

4.1 Double Images

One consequence of a cosmic string's gravitational field is the formation of double images. This is illustrated in figure 1. Suppose a cosmic string (S) is positioned between an observer (O) and a quasar (Q). Light rays passing either side of the string will be bent towards it, and so rays from two different directions can reach the observer. If the line OQ makes an angle ϑ with the string, then the angular separation of the two images is [6, 7]

$$\varphi = \pi\epsilon \frac{L}{L+d} \sin\vartheta \tag{42}$$

where the assumptions $\varphi \ll 1$, $G_4\mu \ll 1$ and $\varphi d \sin\vartheta, \varphi L \sin\vartheta \gg \ell$ have been used. However, in contrast to the result for standard strings, the angle $\pi\epsilon$ is dependent on φ. As $r \to \infty$, where space is flat, the angular momentum satisfies $J = P_T\varphi d/2$. Substituting this into (41) gives an equation for φ. For astronomical distances, $\ell \ll dG_4\mu$, and the angle between the two images is approximately

$$\varphi = \frac{8\pi L}{L+d} G_4\mu \sin\vartheta + \frac{4\ell^2}{\pi d^2 G_4\mu} \frac{L+d}{L} \left(\sin\vartheta + \frac{m^2}{3(E^2 - m^2)\sin\vartheta} \right) \tag{43}$$

The first term is the standard result which arises from the constant part of the deficit angle. The second term gives the brane corrections. For massless particles (e.g. photons) the final energy dependent part is not present.

5 Conclusions

In this paper I have examined the linearized gravitational field around a cosmic string in the Randall-Sundrum brane world scenario. The brane corrections give the string an attractive $1/r^2$ potential, in sharp contrast to standard cosmic strings. At astronomical distances the force is very weak, and so significant gravitational effects will only occur near the string core. Like the standard strings the space at large radii is conical. However as the string core is approached the deficit angle reduces.

Like their non-brane counterparts the brane strings produce double images of objects behind them. The angle between these images will be slightly larger than in the standard case. The closer the light rays pass to the string, the more noticeable the difference. The brane corrections will also alter the CMB anisotropies produced by the string, although I will not consider this here.

The attractive force of the brane string also means that particles can make circular orbits around the string. If the particles also have momentum in the direction parallel to the string, the corresponding paths will be helical. The stability of these orbits is unclear, and depends on gravitational effects beyond the linear approximation. Whether the orbits are stable or not, the string is expected to collect matter as it moves through the universe. This will be in the form of particles which either orbit the string or are pulled into its core.

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Appendix

For the metric (26), the connection coefficients intrinsic to the brane are

$$\hat{\Gamma}^{t}{}_{tr} = \hat{\Gamma}^{z}{}_{zr} = \hat{\Gamma}^{r}{}_{tt} = -\hat{\Gamma}^{r}{}_{zz} = -\frac{1}{2} G_4 \mu \partial_r f_2(r/\ell) \tag{44}$$

$$\hat{\Gamma}^{r}{}_{\theta\theta} = -\partial_r \left[\frac{r^2}{2} (1 - 8G_4\mu\{1 + f_1(r/\ell)\} \right] \tag{45}$$

$$\hat{\Gamma}^{\theta}{}_{r\theta} = \frac{1}{r} - 4G_4\mu\partial_r f_1(r/\ell) \tag{46}$$

The intrinsic Ricci tensor is

$$\hat{R}_{tt} = -\hat{R}_{zz} = \frac{1}{2} G_4 \mu \left[\frac{1}{r} \partial_r f_2(r/\ell) + \partial_r^2 f_2(r/\ell) \right] \tag{47}$$

$$\hat{R}_{rr} = G_4 \mu \frac{1}{r} \partial_r f_2(r/\ell) \tag{48}$$

$$\hat{R}_{\theta\theta} = G_4 \mu r^2 \partial_r f_2(r/\ell) \tag{49}$$

Where the relationship $4xf_1''(x) + 8f_1'(x) + xf_2''(x) + f_2'(x) = 0$ has been used. The intrinsic curvature scalar on the brane is $\hat{R} = 0$.

Using (28) and (29) we see that as $r \to 0$, $\hat{R}_{\mu\nu} \sim \ell/r^3$.

References

[1] A. Vilenkin and E. P. S. Shellard, *Cosmic Strings and other Topological Defects* (Cambridge University Press, Cambridge, England, 1994).

[2] P. de Bernardis *et al.*, Nature **404**, 955 (2000).

[3] N. Turok, U. L. Pen and U. Seljak, Phys. Rev. D **58**, 023506 (1998).

[4] V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. **125B**, 136 (1983);
K. Akama, *Pregeometry* in K. Kikkawa , N. Nakanishi and H. Nariai, eds., *Gauge Theory and Gravitation*, vol. 176 of *Lecture Notes in Physics*, pp. 267-271 (Springer-Verlag, 1983).

[5] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 4690 (1999).

[6] A. Vilenkin, Phys. Rev. D **23**, 852 (1981).

[7] J. R. Gott, Ap. J. **288**, 422 (1985).

[8] N. Kaiser and A. Stebbins, Nature **310**, 391 (1984).

[9] I. Ya. Aref'eva, M. G. Ivanov, W. Mück, K. S. Viswanathan and I. V. Volovich `hep-th/0004114`.

[10] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products* (Academic Press, San Diego, CA, 1980).